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Third Quantization [and Discussion]

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Third quantization

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The problem of describing a system of interacting four-dimensional universes is addressed. String theory provides a soluble model of interacting two-dimensional universes, and suggests a 'third-quantized' description of the four-dimensional problem. Some fascinating consequences of the existence of other universes are discussed within this framework.

It is logically possible, if not intuitively plausible, that the topology of space-time might fluctuate at very short distances. In quantum mechanics almost everything is uncertain, so why should the topology of space-time be absolutely fixed?

String theory is the only known potentially consistent quantum theory of gravity, so it is natural to try to use it to address this question. Space-time topology is fixed in string perturbation theory, but our understanding of non-perturbative string theory is unfortunately far too rudimentary to address the question of whether or not topology fluctuates. In fact, this is an example of a question to which string theory may not provide an answer, even in principle. String theory may be consistent both with and without topology change. Put another way, there are new coupling constants governing the strength of topological fluctuations, and there is no known argument that string theory fixes them.

String theory has, however, been useful in quite another manner in addressing this question. The many possible types of topological fluctuations include a change in the number of connected components of space. This leads us to consider an interacting many-universe system, and there are severe conceptual and practical difficulties in describing such a system. Viewed as a system of interacting two-dimensional universes, string theory provides us with a soluble model of such a system.

In the past year and a half there has been a lot of progress in understanding the consequences of this type of topology change (if it does occur), in part as a result of acquired wisdom from string theory. (For extensive references, and a recent review, see Strominger (1988).) As we shall see, the consequences are quite remarkable and are even potentially observable.

Our starting point for describing topological fluctuations is the euclidean sum-over-four-geometries weighted by the Einstein action. We will be particularly interested in the process illustrated in figure 1 of the nucleation (or annihilation) of a small 'baby' universe by a large 'parent' universe (Strominger 1984; Hawking 1987, 1988; Lavrelashvili *et al.* 1987, 1988; Giddings & Strominger 1988*a*). The rate of this process is obtained in a manner (to be described) from the weighted sum-over-four-geometries with appropriately fixed three geometries on the boundaries.

There are a number of obstacles to evaluating this sum. One problem is that Einstein gravity is non-renormalizable. However, the problem of describing topology fluctuations can in some cases be untangled from the short-distance problems of quantum gravity. This is accomplished

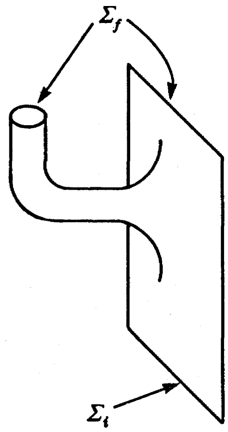

FIGURE 1. A tunnelling process in which the topology of space changes from R^3 on Σ_i to $R^3 \otimes S^3$ on Σ_f.

by regulating the ultraviolet divergences with a short-distance cut-off a, and restricting our attention to cut-off-independent results. This of course will only allow a description of topological fluctuations large relative to the cut-off a.

Another problem is that the euclidean Einstein action is indefinite. This implies that even the regulated functional integral is not convergent. This issue is of great importance, as it has been argued that this divergence explains why the cosmological constant vanishes (Baum 1983; Hawking 1984; Coleman 1988 *a*), but it unfortunately is not well understood at present. It has an analogue in string theory, in that the lorentzian signature of space-time leads to a divergent euclidean world-sheet functional integral. We will assume that here, as in string theory, analytic continuation can be used to define the functional integral.

Even with these assumptions, one cannot of course evaluate the functional integral exactly. What is needed is a small expansion parameter on which to base an approximation scheme. Such a parameter has recently been found (the Peccei–Quinn scale divided by the Planck scale), along with an instanton around which to expand, in a theory of gravity coupled to axions (Giddings & Strominger 1988 *a*). The intractable sum-over-four-geometries is then approximated by a tractable sum-over-instantons. In practice, the expansion around these instantons is our definition of the functional integral describing topology change.

The discovery of these instantons has led to much more concrete discussions of the consequences of baby universes and topology change than was previously possible. One important general feature that has emerged is that baby universes shift space-time coupling constants (Coleman 1988 *b*; Giddings & Strominger 1988 *b*). To see this, consider a small baby universe which contains a positron–electron pair and a photon. The process of it joining a parent universe is illustrated in figure 2 *a*.

It emerges from the instanton calculation that such processes are very unlikely unless the baby universe is very small, of order the Planck length. An observer who cannot make measurements on such small length scales will mistake this process for that of figure 2 *b*. Figure 2 *b* corresponds to a $\bar{\psi}A\psi$ coupling, and a renormalization of the fine structure constant. For example, in a world with no fundamental $\bar{\psi}A\psi$ coupling, one would be induced by the baby universes. Baby universes in general will shift all coupling constants.

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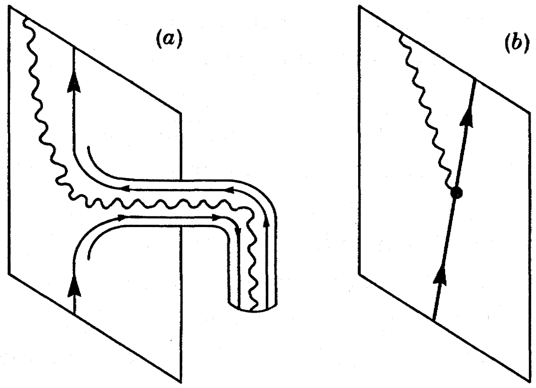

FIGURE 2. (*a*) Interaction with a baby universe carrying a positron, electron (solid lines with arrows) and a photon (wavy line) is indistinguishable at long distances on the parent universe from (*b*) a local $\bar{\psi}A\psi$ coupling.

Let us now be a little more precise about this. The hamiltonian for photons and electrons in the parent universe may be written as

$$H = H_0 + eH_1 + \Delta H_1,$$

where H_0 is the free hamiltonian and

$$H_1 = \int d^3x\bar{\psi}A\psi.$$

ΔH_1 represents the effects from the process of figure 2*a*. These may be described by the introduction of an operator Φ which adds or substracts a baby universe with an electron, a positron and a photon from the bath of baby universes floating around in the void. Φ is referred to as a 'third-quantized' field operator because it creates or annihilates entire second-quantized single-universe states. One then has

$$\Delta H_1 = \Phi H_1.$$

Now suppose (Coleman 1988*b*) that the baby universes are in a coherent state $|\beta\rangle$ such that

$$\Phi|\beta\rangle = \beta|\beta\rangle$$

for some *c*-number β. The operator Φ in H may then be replaced by its eigenvalue β;

$$H = H_0 + (e+\beta)\, H_1.$$

In this case the entire effect of the baby universes is simply to shift the fine structure constant by an amount which depends on the state of the baby universes. This is an example of the important general result

second-quantized coupling constant = third-quantized field eigenvalue.

This result sounds rather strange in this context, but it is actually quite familiar in string theory. It is just the statement that the coupling constants of the two-dimensional string universe are the space-time fields of string field theory.

In string theory, if the two-dimensional coupling constants do not take special values, there will be tadpoles and divergences in the scattering amplitudes. The second-quantized string field will condense and shift the two-dimensional coupling constants so that they obey the string equation of motion. This results in an equation of motion, or dynamical constraint on coupling constants in the two-dimensional string universe. Could the same be true for baby universe induced couplings in our four-dimensional universe?

Before this question can be answered, we must give a more complete description of the dynamics of an interacting many-universe system. The basic process of figure 1 obviously cannot be considered in isolation. Iteration of that process, and accounting for the possibility of baby universes growing up, leads to complicated diagrams as depicted in figure 3. The proper interpretation of these diagrams is by no means obvious. Because each universe has its own time, we certainly can not define dynamics with respect to time on a single universe. We instead proceed by analogy with string theory, and define the system as a quantum field theory on superspace, the space of three geometries (Giddings & Strominger 1988*c*; Banks 1988). (Alternate interpretations are discussed by Hawking & Hartle (1983) and Coleman (1988*a*).) The third-quantized field is a function on superspace, and acts on the 'void' to create entire second-quantized states of a single universe.

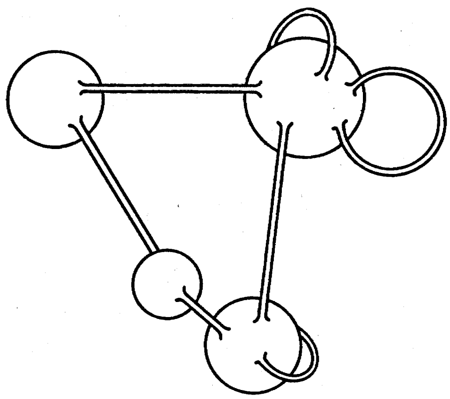

FIGURE 3. In general there can be many parent universes (large spheres) connected in many ways by baby universes (thin tubes).

The action of this third-quantized field theory – a functional of functions on superspace – is defined as the action whose Feynman diagrams reproduce the second-quantized sum-over-four-geometries. This is exactly the route by which correlation functions on the string world sheet are reinterpreted as space-time scattering amplitudes. The analogy is displayed in table 1.

In practice, the third-quantized action has not been constructed. However, in the instanton approximation, the action need only reproduce the sum-over-four-instantons. Rather than quantum field theory on superspace, one then has only quantum field theory on the moduli space of the instanton! In the model of Giddings & Strominger (1988*c*) this space is the real line. The resultant quantum mechanics model can then be constructed quite explicitly.

TABLE 1

string field theory	third-quantized field theory
string	universe
string field	third-quantized field
loop space	superspace of three geometries
$L_0+\tilde{L}_0-2$	Wheeler–De Witt operator
first-quantized BRST charge	second-quantized BRST charge
vacuum	void

In this framework, the third-quantized field, or equivalently the second-quantized space-time couplings, are subject to the third-quantized equation of motion. This constrains the values of the space-time couplings in a manner which would appear quite mysterious to the innocent four-dimensional observer. In the model of Giddings & Strominger (1988*c*), one of these constraints is the vanishing of the cosmological constant (Coleman 1988*a*). Thus the existence of other universes is not only fascinating conceptually, it even has a chance of explaining facets of the universe we inhabit.

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Discussion

J. R. Ellis, F.R.S. (*Theory Division, CERN, Geneva, Switzerland*). Professor Strominger mentioned recent ideas that may lead to a solution of the cosmological constant problem. However, the same ideas seem to lead to problems elsewhere, e.g. the masses of spin-zero particles, such as the pion, should vanish. Is it possible to avoid throwing out the baby with the bathwater?

A. Strominger. It is true that it has been argued by some that the logic that leads to the vanishing of the cosmological constant also leads to the vanishing of the pion mass (while others argue it does not). More generally, there is a very strong singularity that forces the cosmological constant to zero, and there is a danger that all the constants of nature will be dragged to zero (or infinity) along with it. However, what has been done so far should really just be regarded as a first stab at calculating the constants of nature. There are many suspicious assumptions in the calculations of the cosmological constant, and many more in calculations of the pion mass. The physics community is now in the process of scrutinizing these assumptions. We hope to be able to answer this question soon, one way or the other.

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